

Learning Curve

CIMA Terminology: The learning curve is “The mathematical expression of the phenomenon that when complex & labour-intensive procedures are repeated, unit labour times tend to decrease at a constant rate. The learning curve models mathematically this reduction in unit production time.”

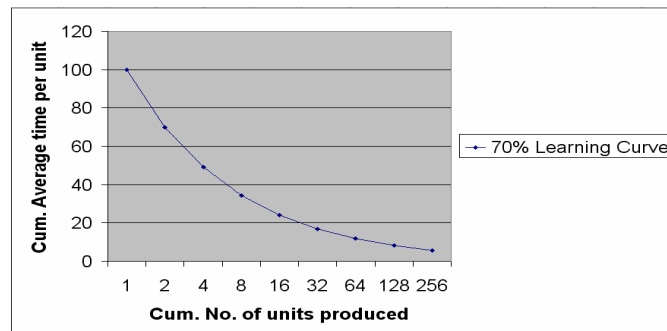
More specifically, the learning curve theory states that the cumulative average time per unit produced is assumed to decrease by a constant percentage every time total output of product doubles. For instance, with an *80% learning curve*, the cumulative average time per unit of output will fall to 80% of what it was before, every time output is doubled.

E.g. If the first unit of output requires 100 hours & an 70% learning curve applies, the production times would be as follows.:

Cumulative Average-Time Learning Model (Doubling Approach)

Cum. No. of units produced	Cum. Average time per unit (Hrs.)	Total time required (Hrs.)	Incremental time taken (Total Hours)	Incremental units produced	Incremental time taken (Hrs. per unit)
1	100.0	100.0 (100*1)	100	1	100 (100÷1)
2 (1*2)	70.0 (100*70%)	140.0 (70*2)	40	1	40 (40÷1)
4 (2*2)	49.0 (70*70%)	196.0 (49*4)	56	2	28 (56÷2)
8 (4*2)	34.3 (49*70%)	274.4 (34.3*8)	78.4	4	19.6 (78.4÷4)

Graph of Learning Curve:



Cumulative Average-Time Learning Model (Equation Approach)

Cum. No. of units produced (X)	Cum. Average time per unit (Hrs.) ¹	Cumulative Total time required (Hrs.)	Incremental time taken/Individual unit time for X th unit
1	100	100	100
2	70	140	40 (140-100)
3	56.8	170.4	30.4 (170.4-140)
4	49	196	25.6 (196-170.4)
5	43.7	218.5	22.5 (218.5-196)
6	39.8	238.8	20.3 (238.8-218.5)
7	36.7	256.9	18.1 (256.9-238.8)
8	34.3	274.4	17.5 (274.4-256.9)

¹The mathematical relationship underlying the cumulative average-time learning model is

$$Y_x = aX^b$$

Where, Y = cumulative average time per unit to produce x units.

X = number of units made so far (cumulative no. of units)

a = time for the first unit

b = the learning coefficient or the index of learning = $\log(\text{learning ratio}) \div \log 2$

(For e.g. for 70% learning curve, $b = \log 0.7 \div \log 2 = -0.1549 \div 0.3010 = -0.5146$)

Learning ratio/percentage = Average labour cost/time of 1st 2N units ÷ Average labour cost/time of 1st N units

- Learning Curve Table gives us factor for learning curve, which is used as value of X^b in the above-mentioned formula of learning curve.
- If the initial output was a batch of units rather than just one, the table of factors would be derived in the same way except that we firstly use factor according to batch & hours calculated on individual units.

Uses of Learning curve:

- CVP Analysis: It helps in analyzing cost-volume profit relationship and is useful for cost estimates and forecasting.
- Budgeting: It helps in preparing realistic budgets and profit planning.
- Pricing: It helps in pricing, particularly in a tender when it is known that the tender consists of several repetitive jobs.
- Decisioning: It helps design engineers in making decision, based upon expected rates of improvement.
- Setting Standards: It helps in setting standards in learning phase.
- Human Resource planning: Its knowledge helps in manpower planning for contract of long duration or for repetitive clerical work.

Limitations of learning curve theory:

1. The learning curve is useful only for new operations where machines do not constitute a major part of the production process. It is not applicable to all productions. E.g. New and experienced workmen.
2. The learning curve assumes that the production will continue without any major interruptions. If for any reason the work is interrupted, the curve may be deflected or assume a new slopes.
3. Changes other than learning may affect the learning curve. For example, improvement in facilities, arrangements, and equipment as well as personnel morale and performance may be factors influencing the curve. On the other hand, negative developments in employee attitudes may also affect the curve and reverse or retard the progress of improvement.
4. The characteristic 80 percent learning curve as originally obtaining in the air force industry in U.S.A. has been usually accepted as the percentage applicable to all industries. Studies show that there cannot be a unique percentage which can be universally applied.
5. Normally, the trade unions will not accept gradual reduction in production time per unit. In this type of situation, management may try to establish a low standard time per unit from the onset. This will lead to adverse efficiency variances, until learning effect has taken place.
6. If there is productivity bonus, incorporation of learning effect may frustrate workers in learning stage, as it may appear as a threat to the size of bonus, which they may earn.

Experience Curve : It is a term applied to the 'corporate embodiment' of the shop floor, managerial & technological learning effects within an organization & it expresses the way in which the average cost per unit of production changes over time due to technological & organizational changes to factory size, product, design, materials used & so on, not just 'learning' by skilled workers.

Important Points in regards to logarithms:

- $Y=x^a$ can also be written as $\log Y = a \log x$
- $\log (x.y) = \log x + \log y$
- $\log (x/y) = \log x - \log y$
- $\log 1 = 0$, $\log 10 = 1$ & $\log 100 = 2$

Question 1: Explain the concept of 'Learning curve'. How can it be applied for Cost Management?

(2 Marks) Nov/06, (4 Marks) May/06 & (4 Marks) May/07

Question 2: Explain the concept of Learning curve and discuss its relevance to pricing decisions.

(4 Marks) May/04

Question 3: What are the distinctive features of learning curve theory in manufacturing environment? Explain the learning curve ratio.

(4 Marks) May/03, (9 Marks) Nov/07

Question 4: In a particular industry, a 60% learning curve applies. The first item of a new product took 100 hours to make. How long should the 6th item take?

[Ans.: 7.51 hours]

Question 5: A company which has developed a new machine has observed that the time taken to manufacture the first machine is 600 hours. Calculate the time which the company will take to manufacture the second machine if the actual learning curve rate is (i) 80% and (ii) 90%. Explain which of the two learning rates will show faster learning. (3 Marks) Nov/08

[Ans.: (i) 360 hours (ii) 480 hours; 80% is shows faster learning]

Question 6: ABC Co. Ltd. has observed that a 80% learning curve ratio applies to all labour-related costs realating Model X, which has been recently introduced in the market. It is expected that first unit will take a time of 10 minutes. What is the expected time per unit (i) when cumulative production is 20 units, and (ii) when cumulative production is 10 units?

[Ans.: (i) 3.812 minutes & (ii) 4.764 minutes]

Question 7: (a) Your company has been approached by a customer to supply four units of a new product made to the customer's individual specification. The company experiences a 90% learning rate. The estimated labour time for the first unit of this product of this product is 150 hours and the company's direct labour cost is Rs. 5 per hour. Estimate the labour cost for this order.

(b) After receiving the first order, if the customer places a repeat order, what will be the labour cost for the second order.

(c) If the customer had ordered all eight units at the same time, calculate the labour cost per unit for the combined order.

[Ans.: (a) 2430 (b) 1944 (c) 4374]

Question 8: The 1st batch of 50 units of Product A took 80 hours to make. The company now wishes to estimate, what average time per unit will be if the total output for product A is 125 units. An 80% learning curve applies.

[Ans.: 1.2 hours]

Question 9: A customer requires separate price quotations for each of the following possible order:

Order	Units
First	100
Second	60
Third	40

It is expected that in first order 4 hours per unit will be consumed. A 80% learning curve applies. Following learning curve table for 80% learning curve is given:

X	1.0	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
Y%	100.0	91.7	89.5	87.6	86.1	84.4	83.0	81.5	80.0

Find out average time per unit:

(a) when two orders are expected.

(b) When three orders are expected.

[Ans.: (a) 3.44 (b) 3.2]

Question 10: M Ltd. manufactures a special product purely carried out by manual labour. It has a capacity of 20000 units. It estimates the following cost structure:

Direct Material	30 Rs./unit
Direct Labour (1 hour/unit)	20 Rs./unit
Variable overhead	10 Rs./unit

Fixed overheads at maximum capacity are Rs. 150000.

It is estimated that at the current level of efficiency, each unit requires one hour for the first 5000 units. Subsequently it is possible to achieve 80% learning rate. The market can absorb the first 5000 units at Rs. 100 per unit. What should be the minimum selling price acceptable for an order of 15000 units for a prospective client? (7 Marks) May/08

[Ans.: Min. Selling Price is Rs. 50.40]

Question 11: ABC Ltd. is a company engaged in the provision of small quantities of specialized equipment to industrial customers. The work involved in the manufacture of the output calls for a high level of manual skill, and since no product has a long technological life, it is thought that a 70% learning curve applies to all output made by the company.

A customer has asked for the delivery of eight units of a product which has never before been made. An estimate of the direct cost of the first unit has been derived as follows:

Materials	Rs. 400
Direct labour (250 hours)	<u>1250</u>
	<u>1650</u>

The company adds a contribution margin of 160% on direct cost to cover fixed costs and provide a profit margin.

Required:

- State the formula for the 70% learning curve; and
- Use the formula to calculate the estimated direct costs and sales prices of the ten units of product.

[Ans.: 2033.59 per unit]

Question 12: A company has 10 direct workers, who work for 25 days a month of 8 hours per day. The estimated down time is 25% of the total available time. The company received an order for a new product. The first unit of the new product requires 40 direct labour hours to manufacture the product. The company expects 80% (index is -0.322) learning curve for this type of work. The company uses standard absorption costing and the cost data are as under:

Direct materials	Rs. 60 per unit
Direct labour	Rs. 6 per direct labour hour
Variable overheads	Rs. 1 per direct labour hour
Fixed overheads	Rs. 7500 per month

Required:

- Calculate the cost per unit of the first order of 30 units.
- If the company receives a repeat order for 20 units, what prices to be quoted to yield a profit of 25% on selling price? (8 Marks) Nov/02 & CIMA(London)-Adapted

[Ans.: (i) 220.56 (ii) 159.66]

Question 13: ABC Ltd. has designed a new type of sailing boat, for which the cost and sales price of the first boat to be produced has been estimated as follows:

Materials	Rs. 5000
Labour (800 hrs. X Rs. 5 per hour)	<u>4000</u>
Overhead (150% of labour cost)	<u>6000</u>
	<u>15000</u>
Profit mark-up (20%)	<u>3000</u>
Sales price	<u>18000</u>

It is planned to sell all the yachts at full cost plus 20%. An 80% learning curve is expected to apply to the production work. A customer interest in buying the yachts, but thinks Rs. 18000 is too high price to pay. He might want to buy 2, or even 4 of the yachts during the next six months.

He has asked the following questions:

- If he paid Rs. 18000 for the first yacht, what price would he have to pay later for the second yacht?
- Could ABC Ltd. quote same unit price for two yachts, if the customer ordered two at the same time?
- If the customer bought two yachts now at one price, what would be the price per unit for a third and a fourth yacht, if he ordered them separately later on?
- Could ABC Ltd. quote a single unit price for (i) four yachts; (ii) eight yachts; if they were all ordered now.

Assuming that there are no other prospective customers for the yachts, how would these questions be answered?

[Ans.: (a) 13200 (b) 15600 (c) 11760 (d) 13680 and 12144]

Question 14: An electronics firm which has developed a new type of fire-alarm system has been asked to quote for a prospective contract. The customer requires separate price quotations for each of the following possible orders:

Order	Number of fire-alarm systems
First	100

Second	60
Third	40

The firm estimates the following cost per unit for the first order:

Direct materials	Rs. 500
Direct labour	
Deptt. A (Highly automatic) 20 hours at Rs. 10 per hour	
Deptt. B (Skilled labour) 40 hours at Rs. 15 per hour	
Variable overheads	20% of direct labour
Fixed overheads absorbed:	
Deptt. A	Rs. 8 per hour
Deptt. B	Rs. 5 per hour

Determine a price per unit for each of the three orders, assuming the firm uses a mark up of 25% on total costs and allows for an 80% learning curve. Extract from 80% Learning curve table:

X	1.0	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
Y%	100.0	91.7	89.5	87.6	86.1	84.4	83.0	81.5	80.0

X represents the cumulative total volume produced to date expressed as a multiple of the initial order. Y is the learning curve factor, for a given X value, expressed as a percentage of the cost of the initial order.

(8 Marks) May/05, ICWA Nov/05 & CIMA – Adapted

[Ans.: (i) Price (P.V) of 1st order (100 units) = 2275 (ii) (P.V) of 2nd order (60 units) = 1848.64 (iii) (P.V) of 3rd order (40 units) = 1764.40]

Question 15: A company has accepted an order for making 15 items of a specialized machine at a price of Rs. 4 lacs each. The delivery is to be completed within 4 months. The company works 23 days a month and the normal direct wages per day amounts to Rs. 10000. However, in case of need, the company can work overtime up to 8 days during the said period at double the normal rate of wages. Overheads amount to Rs. 12000 per normal working day but no overheads are charged on overtime working days. The material cost is Rs. 240000 per machine. The company has estimated that it will take 10 working days to manufacture the first machine. The company is expected to experience a learning effect of 90% ($b=0.152$). The contract stipulates a penalty of Rs. 40000 per machine delivered beyond the schedule of 4 months.

You are required to calculate the costs and advise the company whether it is preferable to work only during normal working days and pay penalty for any delayed delivery of the machines or to work overtime to avoid paying penalty. (ICWA June, 2000)

[Ans.: Additional profit is Rs. 6000]

Question 16: A firm has developed a product for which the following standard cost estimates have been made for first batch to be manufactured in Month 1:

Standard Cost for the batch	
500 labour hours @ 8 per hour	Rs. 4000
55 units of direct materials @ Rs. 100 per unit	5500
Variable overhead 500 hours @ 15 per hour	7500
	<u>17000</u>

From experience the firm knows that labour will benefit from a learning benefit and labour time will be reduced. This is expected to approximate to an 80% learning curve and to follow the general function.

$$y = ax^b$$

Where y = average labour hours per batch
 a = number of labour hours for the first batch
 x = cumulative number of batches
and b = learning coefficient.

(The learning coefficient is found as follows:

$$b = \log (1 - \text{proportionate decrease}) \div \log 2$$

The coefficient for an 80% learning curve is $b = -0.322$.)

In addition, the growing expertise of labour is expected to improve the efficiency with which materials are used. The usage of materials is expected to approximate to a 95% learning curve and to follow the general function.

$$y = ax^b$$

Where y = average labour hours per batch
 a = number of labour hours for the first batch
and x and b are explained previously.

The actual production for the first six months was as follows:

Month 1	20 batches	Month 4	24 batches
Month 2	30 batches	Month 5	33 batches
Month 3	25 batches	Month 6	28 batches

During Month 6 the following results were recorded for the last batch made:

Actual results of last batch

Labour hours	115
Direct wages	Rs. 978
Direct materials (41 units)	3977
Variable overhead	1685

You are required:

- (a) to calculate the learning coefficient for materials;
- (b) to derive the Standard Cost of the last batch in Month 6;
- (c) to calculate what variances have arisen in connection with the last batch;
- (d) to explain what information the variances provide for management.

[Ans.: (a) -0.074 (b) Rs. 6021 (c) LRV = 58V, LEV = 140A, MPV = 123F, MVV = 322A, VOExp. V = 40F, VOEff. V = 262A]

Please go through concepts of Absorption Costing before doing Question 12 and Question 15, through Standard Costing before doing Question 16, & through Direct Costing before doing Question 10

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	Mean Differences								
											1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5808	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6514	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	3	4	5	6	6	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7083	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	3	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6

	0	1	2	3	4	5	6	7	8	9	Mean Differences								
											1	2	3	4	5	6	7	8	9
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	1	1	2	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9547	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9982	9917	9921	9928	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	Mean Differences								
											1	2	3	4	5	6	7	8	9
.00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
.01	1023	1026	1018	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
.02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
.03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
.04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
.05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
.06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
.07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
.08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
.09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
.10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
.11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
.12	1318	1321	1324	1327	1330	1333	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
.13	1349	1352	1355	1358	1361	1364	1368	1371	1374	1377	0	1	1	1	2	2	2	2	3
.14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	2	3
.15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	2	3
.16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
.17	1479	1483	1486	1489	1492	1493	1496	1500	1503	1507	0	1	1	1	2	2	2	3	3
.18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
.19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	2	3	3
.20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	2	3	3
.21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
.22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
.23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
.24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
.25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
.26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
.27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
.28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
.29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
.30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
.31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
.32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
.33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
.34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
.35	2239	2245	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
.36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
.37	2344	2350	2355	2360	2366	2371	2376	2381	2386	2393	1	1	2	2	3	3	4	4	5
.38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
.39	2455	2460	2465	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	4	5
.40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
.41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
.42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
.43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
.44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
.45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
.46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
.47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
.48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
.49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6
.50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	8
.51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	8
.52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	8

	0	1	2	3	4	5	6	7	8	9	Mean Differences								
											1	2	3	4	5	6	7	8	9
.53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	8
.54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	8
.55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	8
.56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
.57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
.58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
.59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
.60	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
.61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
.62	4169	4178	4188	4198	4207	4217	4227	4236	4256	4256	1	2	3	4	5	6	7	8	9
.63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
.64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
.65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
.66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
.67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
.68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
.69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
.70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
.71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
.72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
.73	5376	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
.74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	11
.75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	11
.76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
.77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
.78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
.79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
.80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
.81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
.82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
.83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
.84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
.85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
.86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
.87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
.88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
.89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
.90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
.91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
.92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
.93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
.94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
.95	8913	8933	8954	8974	8895	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
.96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
.97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
.98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
.99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20